Operations, analysis and fractals without complex numbers

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October 31, 2025

Abstract

This paper is a continuation of viXra:2508.0176, in which we saw that we can avoid the concepts of negative number and complex number thanks to the study of the underlying vector nature of some arithmetic and polynomial problems. With the solutions of the polynomial equations which were actually geometrical, in the Euclidean vector space, we will construct several operations which are analogous to what we have seen until now with "complex numbers". We will show also the representations of functions whose arguments are vectors. We will see the basic elements needed in order to rebuild all what has been constructed in complex analysis. We will show also that we can construct the Mandelbrot set in the Euclidean vector space.

1 Introduction

Complex numbers appeared when some mathematicians tried to solve polynomial equations of degree 3 during the XVIth century. Between the end of the XVIIIth century and the beginning of the XIXth century, the discovery of the geometrical interpretations of complex numbers lead to the complex plane and to the complex analysis (see [Ahlfors1979]). Some fractals were discovered by mathematicians as Gaston Julia and Benoît Mandelbrot (see [WikipediaM]).

In ViXra:2508.0176 we saw that we don't need the concept of complex number nor the concept of negative number. In this paper we will see that the geometrical solutions that we found for several problems can be used to perform several operations. We will see also functions whose arguments are those geometrical solutions applied to a unit vector.

2 The plane of all the images of the horizontal unit vector

We have seen (see [Torres-Heredia2025]) that the solutions of the problems that were actually geometrical have the following form :

$$X = H_\lambda \circ R(\theta)$$

We saw that this solution is linked to the unit vector $\begin{pmatrix}1\\0\end{pmatrix}$, so we can see the effect of that solution on the unit vector:

$$X(\begin{pmatrix}1\\0\end{pmatrix}) = H_\lambda \circ R(\theta)(\begin{pmatrix}1\\0\end{pmatrix})$$

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The result of the operation on the unit vector is another vector:

$$X(\begin{pmatrix}1\\0\end{pmatrix}) = \lambda \begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix}$$

$$\begin{aligned} X(\begin{pmatrix}1\\0\end{pmatrix}) &= \lambda \begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix} \\ &= \begin{pmatrix}\lambda\cos(\theta)\\ \lambda\sin(\theta)\end{pmatrix} \end{aligned}$$

Now, let $a = \lambda\cos(\theta)$ and $b = \lambda\sin(\theta)$. So,

$$\begin{aligned} X(\begin{pmatrix}1\\0\end{pmatrix}) &= \begin{pmatrix}a\\b\end{pmatrix} \\ &= a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix} \end{aligned}$$

And that corresponds to what was considered until now as a complex number: $a + ib$.

So, it is clear that if $\lambda \in \mathbb{R}_+$ and if θ can take all values, the set of all the possibles images of the unit vector by all the operations X will be the plane $\mathbb{R}^2$ or the Euclidean plane $\mathbb{E}^2$.

Now, we can consider the Euclidean plane $\mathbb{E}^2$ as the result of all the possible results of operations consisting in a composition of a homothety and a rotation on the unit vector. It is always the same plane but each vector will be considered as the result of a rotation and a homothety on the unit vector.

If we take, for exemple, the vector $\begin{pmatrix}3\\4\end{pmatrix}$, we will consider it as the result of the composition of a rotation and a homothety on the unit vector. And we can retrieve that composition:

$$\lambda = \sqrt{3^2 + 4^2}$$
$$\theta = \arctan(\frac{4}{3})$$

So we will have:

$$\lambda = 5$$
$$\theta \approx 53.13^\circ$$

So that composition is:

$$H_5 \circ R(53.13^\circ)$$

So each vector will be seen in relation to the unit vector $\begin{pmatrix}1\\0\end{pmatrix}$ (see the figure 1)

By the way, as we have seen before (see [Torres-Heredia2025]), we can avoid the notion of negative number thaks to rotations of 180° of the unit vector $\begin{pmatrix}1\\0\end{pmatrix}$. But, for pratical reasons, in the following sections we will use the négative labels as "-2" with the usual operations.

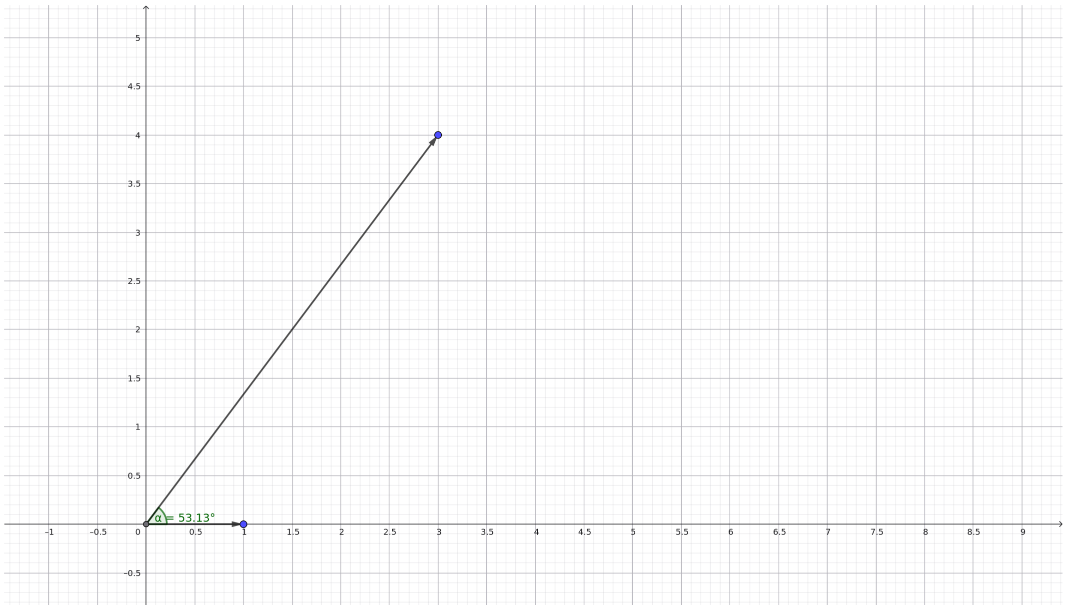

Figure 1: A vector in relation to the horizontal unit vector

3 The 5 forms of the mathematical object

In the following sections we will use 5 forms of the mathematical object we are studying.

The first one is the geometrical operation linked to the unit vector $\begin{pmatrix}1\\0\end{pmatrix}$:

$$X = H_\lambda \circ R(\theta)$$

The second one is the operation applied to that vector:

$$X(\begin{pmatrix}1\\0\end{pmatrix}) = H_\lambda \circ R(\theta)(\begin{pmatrix}1\\0\end{pmatrix})$$

The third one is the operation applied to that vector using a rotation matrix:

$$\lambda R(\theta)\begin{pmatrix}1\\0\end{pmatrix}$$

The fourth one is the vector version

$$\begin{pmatrix}a\\b\end{pmatrix}$$

where:

$$\begin{pmatrix}a\\b\end{pmatrix} = \lambda R(\theta)\begin{pmatrix}1\\0\end{pmatrix}$$

The fifth one is another vector version linked to angles:

$$\begin{pmatrix}a\\b\end{pmatrix} = \begin{pmatrix}\lambda\cos(\theta)\\\lambda\sin(\theta)\end{pmatrix}$$

The second, the third and the fifth forms correspond to what we called the polar form of a complex number.

4 The operations with the mathematical objects and the resulted vectors of these operations

4.1 The addition

In the plane seen before, which is no other than $\mathbb{E}^2$, we can perform additions of vectors with the usual rule:

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+b \\ c+d \end{pmatrix}$$

4.2 The composition and the new multiplication

As we have seen before, each vector of that plane is the result of the composition of a rotation and a homothety applied to the unit vector. If we have two vectors, we can compose each one of their associated compositions in order to have a third composition of a rotation and a homothety.

For example, if we have two vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$, we can retrieve their associated compositions as seen before, $H_\lambda \circ R(\theta)$ and $H_\mu \circ R(\varphi)$. If we compose those 2 operations, we will get $H_{\mu\lambda} \circ R(\theta+\varphi)$. Then, we can apply this new composition to the unit vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and we will get a third vector $\begin{pmatrix} e \\ f \end{pmatrix}$.

We can also perform a calculation, using the two vectors and the rotations of the unit vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$. We can express the two vectors in terms of that unit vector, where $R(90°)$ is a rotation matrix:

$$\begin{aligned} \begin{pmatrix} a \\ b \end{pmatrix} &= a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ &= a \begin{pmatrix} 1 \\ 0 \end{pmatrix} + bR(90°) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned}$$

and, by the same way:

$$\begin{pmatrix} c \\ d \end{pmatrix} = c \begin{pmatrix} 1 \\ 0 \end{pmatrix} + dR(90°) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Then we rewrite the 2 vectors as this, where I_2 stands for the identity matrix of size 2:

$$\begin{aligned} \begin{pmatrix} a \\ b \end{pmatrix} &= aI_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + bR(90°) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} c \\ d \end{pmatrix} &= cI_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + dR(90°) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{aligned}$$

Now, thanks to the rules of distribution of matrix, we get:

$$
\begin{aligned}
\begin{pmatrix} a \\ b \end{pmatrix} &= (aI_2 + bR(90°)) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
\begin{pmatrix} c \\ d \end{pmatrix} &= (cI_2 + dR(90°)) \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\end{aligned}
$$

So, we have expressed the compositions of rotations and homotheties associted to the 2 vectors in terms of matrix and the numbers a,b,c and d.

Now we can compose those two operations thanks to matrix multiplications, using the rules of matrix distribution:

$$
\begin{aligned}
(aI_2 + bR(90°))(cI_2 + dR(90°)) &= aI_2cI_2 + aI_2dR(90°) + bR(90°)cI_2 + bR(90°)dR(90°) \\
&= acI_2 + adR(90°) + bcR(90°) + bdR(180°) \\
&= acI_2 + (ad + bc)R(90°) + bdR(180°)
\end{aligned}
$$

And now we can apply this operation to unit vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$:

$$
\begin{aligned}
(acI_2 + (ad+bc)R(90°) + bdR(180°)) \begin{pmatrix} 1 \\ 0 \end{pmatrix} &= acI_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (ad+bc)R(90°) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + bdR(180°) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\
&= ac \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (ad+bc) \begin{pmatrix} 0 \\ 1 \end{pmatrix} + bd \begin{pmatrix} -1 \\ 0 \end{pmatrix} \\
&= (ac - bd) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (ad+bc) \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\end{aligned}
$$

So, if we compose the associated compositions of rotations and homotheties of the vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$, we get a new compostion of a rotation and a homothety. And if we apply this new composition to the unit vector, we get a third vector:

$$
\begin{aligned}
\begin{pmatrix} e \\ f \end{pmatrix} &= (ac - bd) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (ad+bc) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} ac - bd \\ ad + bc \end{pmatrix}
\end{aligned}
$$

What we have done corresponds to the multiplication of two complex numbers:

$$
(a + ib)(c + id) = (ac - bd) + i(ad + bc)
$$

As we have seen, we have performed multiplications of matrix to get our result. For practical reasons, we can define a new multiplication between vectors, called $*$ in this framework:

$$\begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac - bd \\ ad + bc \end{pmatrix}$$

So, given 2 vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$ with their associated compositions of rotations and homotheties, as seen before, the operation $\begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} c \\ d \end{pmatrix}$ will give us a third vector. As we have seen before, if we compose the associated compositions of rotations and homotheties of the vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$, we get a new compostion of a rotation and a homothety. And if we apply this new composition to the unit vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, we get a third vector. And this third vector is the result of $\begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} c \\ d \end{pmatrix}$.

And this third vector will have an associated composition of a homothety and a rotation $H_{\mu\lambda} \circ R(\theta + \varphi)$.

4.3 The inverse of a vector

Given a vector $\begin{pmatrix} a \\ b \end{pmatrix}$, we look for a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ such that

$$\begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

So,

$$\begin{pmatrix} ax - by \\ ay + bx \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

To continue, we must solve a system of linear equations:

$$\begin{cases} ax - by = 1 \\ ay + bx = 0 \end{cases}$$

The wxMaxima Algebra System gives us:

$$\left[x = \frac{a}{b^2 + a^2}, y = -\frac{b}{b^2 + a^2}\right]$$

So,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{a}{a^2+b^2} \\ -\frac{b}{a^2+b^2} \end{pmatrix}$$

Now, we can define the inverse of a vector in this framework. If $\begin{pmatrix} a \\ b \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, then:

$$\begin{pmatrix} a \\ b \end{pmatrix}^{-1} = \begin{pmatrix} \frac{a}{a^2+b^2} \\ -\frac{b}{a^2+b^2} \end{pmatrix}$$

4.4 The division of two vectors

Given 2 vectors $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$, if $\begin{pmatrix} c \\ d \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, we define in this framework:

$$\frac{\begin{pmatrix} a \\ b \end{pmatrix}}{\begin{pmatrix} c \\ d \end{pmatrix}} = \begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} c \\ d \end{pmatrix}^{-1}$$

So,

$$\begin{aligned} \frac{\begin{pmatrix} a \\ b \end{pmatrix}}{\begin{pmatrix} c \\ d \end{pmatrix}} &= \begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} \frac{c}{c^2+d^2} \\ -\frac{d}{c^2+d^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{ac}{c^2+d^2} + \frac{bd}{c^2+d^2} \\ -\frac{ad}{c^2+d^2} + \frac{bc}{c^2+d^2} \end{pmatrix} \\ &= \begin{pmatrix} \frac{ac+bd}{c^2+d^2} \\ \frac{-ad+bc}{c^2+d^2} \end{pmatrix} \end{aligned}$$

And knowing that:

$$\begin{pmatrix} a \\ b \end{pmatrix} = \lambda R(\theta) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and that:

$$\begin{pmatrix} c \\ d \end{pmatrix} = \mu R(\varphi) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and knowing also, as we have seen before, that the multiplication of two vectors involves the addition of their associated angles and the multiplication of the numbers which define their associated homotheties, we will have now a subtraction and a division:

$$\frac{\begin{pmatrix} a \\ b \end{pmatrix}}{\begin{pmatrix} c \\ d \end{pmatrix}} = \frac{\lambda}{\mu} R(\theta - \varphi) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

4.5 The exponentiation

We can define a new exponentiation if we multiply a vector by himself several times:

$$\begin{pmatrix} a \\ b \end{pmatrix}^n = \underbrace{\begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} a \\ b \end{pmatrix} * \cdots * \begin{pmatrix} a \\ b \end{pmatrix} * \begin{pmatrix} a \\ b \end{pmatrix}}_{n\ times}$$

And knowing that:

$$\begin{pmatrix} a \\ b \end{pmatrix} = \lambda R(\theta) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and knowing also, as we have seen before, that the multiplication of two vectors involves the addition of their associated angles and the multiplication of the numbers which define their associated homotheties, we will have:

$$\begin{pmatrix} a \\ b \end{pmatrix}^n = \lambda^n R(n\theta) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

4.6 The roots

Given a vector $\begin{pmatrix} a \\ b \end{pmatrix}$, we have seen that we can retrive its associated composition of a rotation and a homothety applied to the unit vector $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$:

$$\begin{pmatrix} a \\ b \end{pmatrix} = H_\lambda \circ R(\theta)(\begin{pmatrix} 1 \\ 0 \end{pmatrix})$$

As we have seen before, we can use also a matrix to express the rotation, so:

$$\begin{pmatrix} a \\ b \end{pmatrix} = \lambda R(\theta) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

where:

$$\lambda = \sqrt{a^2 + b^2}$$
$$\theta = \arctan(\frac{b}{a})$$

As we have seen before, if we multiply two vectors with the new multiplication used in this framework, the third vector will be associated to an addition of the angles associated to the operands and will be also associated to the mutiplication of the numbers which define the homotheties associated to the operands.

So, if we multiply a vector by himself, the result will be associated to an angle equal to the double of the angle associated to the first vector and it will be also associated to the square of the number which define the homothety associated to the first vector. If we multiply a vector by himself 3 times, the result will be associated to an angle equal to the triple of the angle associated to the first vector, and it will be also associated to the cube of the number which define the homothety associated to the first vector.

So, in order to find the roots of a vector in this framework, we must to work with divisions of the angle associated to the vector and we must find the roots of the number which define the homothetiy of the vector.

So, for example, a square root of $\begin{pmatrix} a \\ b \end{pmatrix}$ will be:

$$H_{\sqrt{\lambda}} \circ R(\frac{\theta}{2})(\begin{pmatrix} 1 \\ 0 \end{pmatrix})$$

And using a rotation matrix we have:

$$\sqrt{\lambda}R(\frac{\theta}{2})\begin{pmatrix}1\\0\end{pmatrix}$$

But, as we are working with angles and sinus and cosinus, as we have seen before, there will be other solutions linked to the resolution of trigonometric equations. In fact, in order to find all the roots, we must to solve the following equation, where $\vec{x} = \begin{pmatrix}x_1\\x_2\end{pmatrix}$:

$$\vec{x}^n = \begin{pmatrix}a\\b\end{pmatrix}$$

or:

$$\begin{pmatrix}x\\y\end{pmatrix}^n = \lambda R(\theta)\begin{pmatrix}1\\0\end{pmatrix}$$

We know also that:

$$\begin{pmatrix}x\\y\end{pmatrix} = \mu R(\phi)\begin{pmatrix}1\\0\end{pmatrix}$$

And so,

$$\vec{x}^n = \mu^n R(n\phi)\begin{pmatrix}1\\0\end{pmatrix}$$

We can write also, as seen before (see section 3, page 3):

$$\vec{x}^n = \mu^n \begin{pmatrix}\cos(n\phi)\\ \sin(n\phi)\end{pmatrix}$$

And also:

$$\begin{pmatrix}a\\b\end{pmatrix} = \lambda\begin{pmatrix}\cos(\theta)\\ \sin(\theta)\end{pmatrix}$$

Then we must to solve this equation:

$$cos(n\phi) = cos(\theta)$$

So,

$$\phi = \frac{\theta + k2\pi}{n}$$

where $k = 1, 2, 3, ..., n-1$ because when $k \geq n$, we get in fact angles which will produce the same vectors.

And then we have the general form of all the n roots:

$$\vec{x}_k = \sqrt[n]{\lambda}R(\frac{\theta + k2\pi}{n})\begin{pmatrix}1\\0\end{pmatrix}$$

5 The field structure of $\mathbb{E}^2$ with the new operations

We can show that $\mathbb{E}^2$, together with the 2 operations $+$ and $*$ defined in this framework, is a field. The additive identity is the vector $\begin{pmatrix}0\\0\end{pmatrix}$ and the multiplicative identity is is the vector $\begin{pmatrix}1\\0\end{pmatrix}$.

6 The functions and their representations

With the operations that we have defined, we can construct functions. For example:

$$f(\vec{x}) = (\vec{x} + \begin{pmatrix}1\\0\end{pmatrix})^2$$

where $\vec{x} = \begin{pmatrix}x_1\\x_2\end{pmatrix}$. So,

$$\begin{aligned}
f(\vec{x}) &= (\vec{x} + \begin{pmatrix}1\\0\end{pmatrix}) * (\vec{x} + \begin{pmatrix}1\\0\end{pmatrix}) \\
&= \vec{x}^2 + 2\vec{x} * \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}^2 \\
&= \vec{x} * \vec{x} + 2\vec{x} * \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix} * \begin{pmatrix}1\\0\end{pmatrix} \\
&= \vec{x} * \vec{x} + 2\vec{x} + \begin{pmatrix}1\\0\end{pmatrix} \\
&= \begin{pmatrix}x_1^2 - x_2^2\\2x_1x_2\end{pmatrix} + \begin{pmatrix}2x_1\\2x_2\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix} \\
&= \begin{pmatrix}x_1^2 - x_2^2 + 2x_1 + 1\\2x_1x_2 + 2x_2\end{pmatrix}
\end{aligned}$$

So this is function defined from $\mathbb{E}^2$ to $\mathbb{E}^2$, $f : \mathbb{E}^2 \to \mathbb{E}^2$.

In order to represent graphically this function, there are two ways. The first one consist in representing firstly a subset of $\mathbb{E}^2$ and then in representing its image. For example, if we have the unit circle as a subset (see the figure 2), its image will be another set of points (see the figure 3), which is a kind of cardioid. Obviously, we represent only the terminal points of each vector.

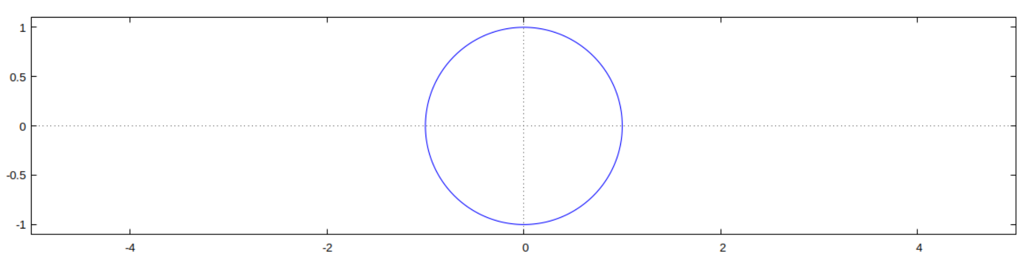

Figure 2: The unit circle as a subset

The other way to represent graphically this function is to extract 2 other functions from $\mathbb{R}^2$ to $\mathbb{R}$. Those 2 functions are built from the x and y components of the result of the function f. So we will have:

$$f_x(x_1, x_2) = x_1^2 - x_2^2 + 2x_1 + 1$$

and

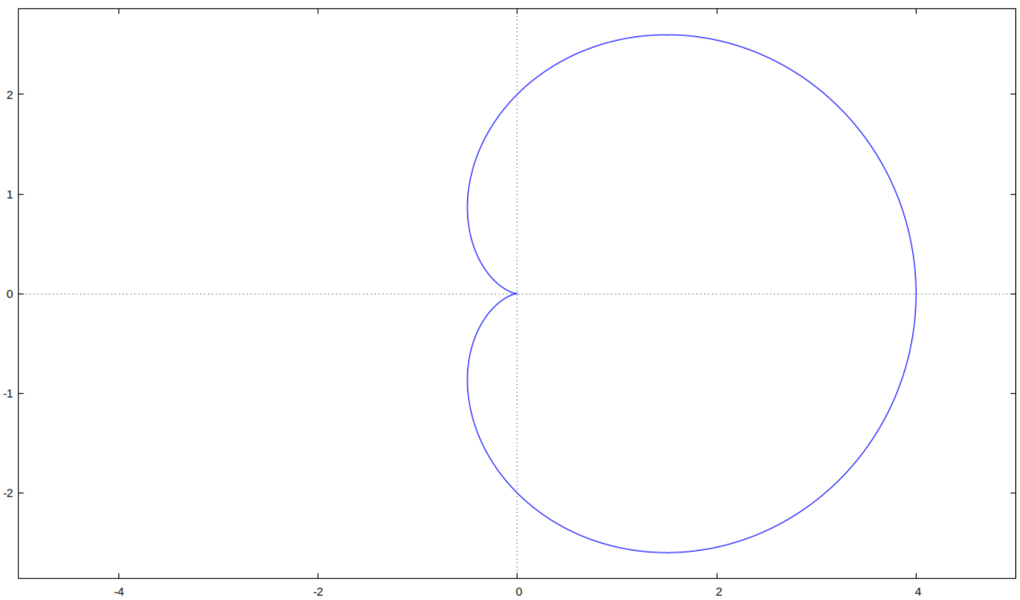

Figure 3: The image of unit circle by the function f

$$f_y(x_1, x_2) = 2x_1x_2 + 2x_2$$

And now we can represent them graphically (see the figure 4 and the figure 5). In fact, f_x corresponds to the $Re(f(z))$ of the complex functions and f_y corresponds to the $Im(f(z))$ (see [GlaeserPolthier2013], p. 288).

With this construction of functions, we can continue to develop limits, derivatives, etc. in an analogous ways it has be done in complex analysis (see [Ahlfors1979]).

7 Polynomials

Obviously, the mathematical objects we are studying come from polynomial equations (see [Torres-Heredia2025]). But what is new here is that we can construct also polynomials with the vectors and the new operations defined in $\mathbb{E}^2$. So we can have a polynomial as:

$$\vec{P}(\vec{x}) = \vec{a_0} + \vec{a_1} * \vec{x} + \vec{a_2} * \vec{x}^2 + \vec{a_3} * \vec{x}^3 + \vec{a_4} * \vec{x}^4$$

So $\vec{P}$ is also a vector. And we can find the roots of this polynomial with several methods.

This polynomial corresponds to what we called complex polynomials, with complex coefficients.

8 The Mandelbrot Set

Now we can construct the Mandelbrot set (see [WikipediaM]) only with vectors and the new operations of this framework in $\mathbb{E}^2$.

Let $\vec{c} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$ and $\vec{x_0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

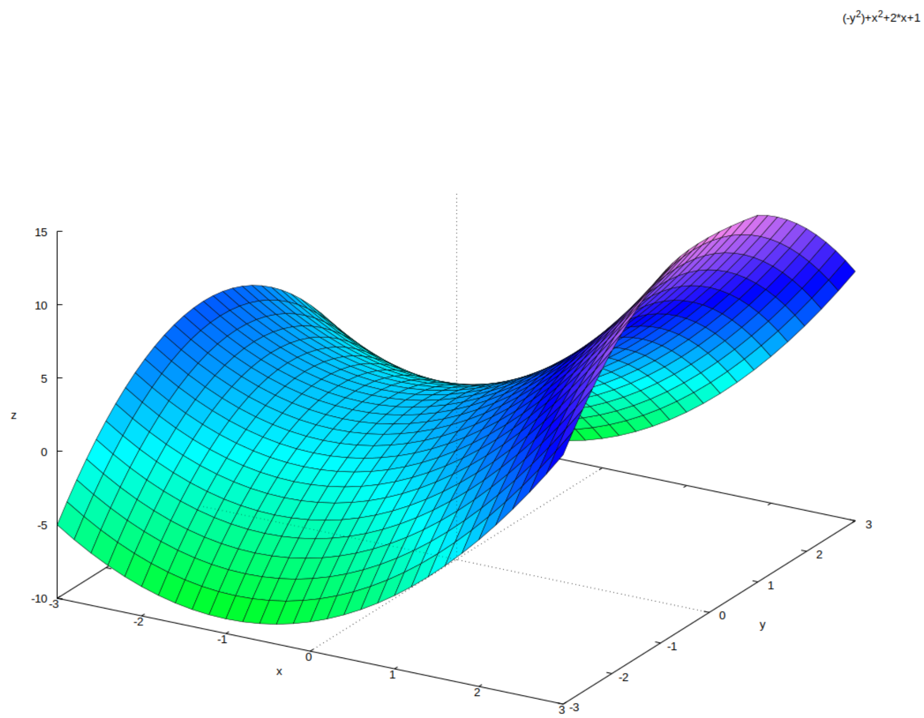

Figure 4: The function corresponding to the x component of f

We will perform an iteration of a quadratic map to get the vectors $\vec{x_k}$:

$$\vec{x_k} = \vec{x_{k-1}}^2 + \vec{c}$$

And we will see if the vector norm of $\vec{x_k}$ remains bounded for all $k > 0$. If it is the case, we will add the vector $\vec{c}$ to the Mandelbrot set.

So, we must to apply that iteration to several values of $\vec{c}$ and we will have the complete Mandelbrot set in $\mathbb{E}^2$. We can represent graphically this set (see the figure 6). Obviously, we represent only the terminal points of each vector $\vec{c}$.

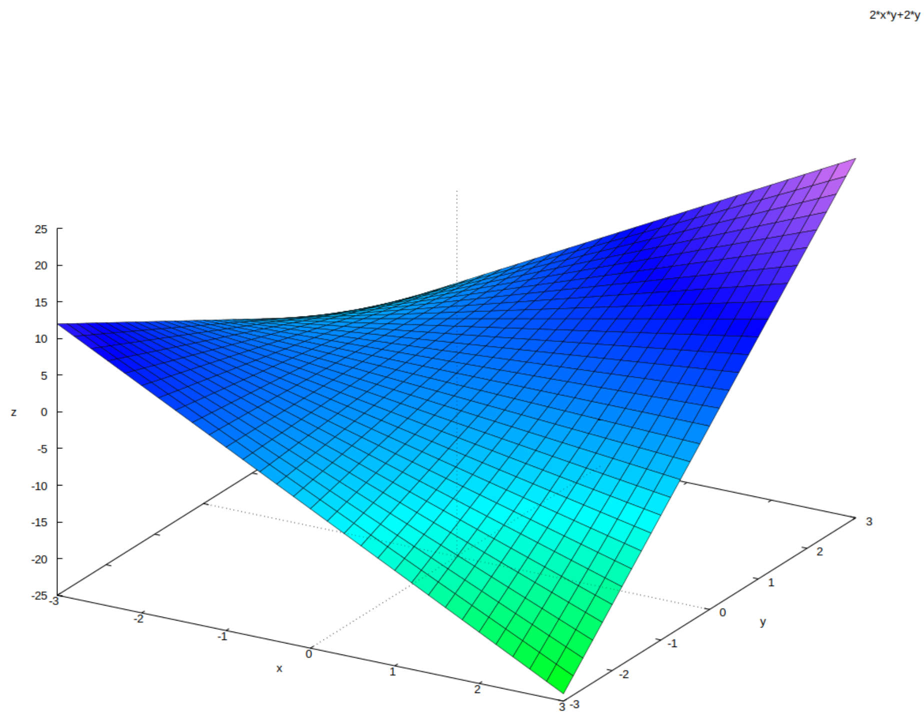

Figure 5: The function corresponding to the y component of f

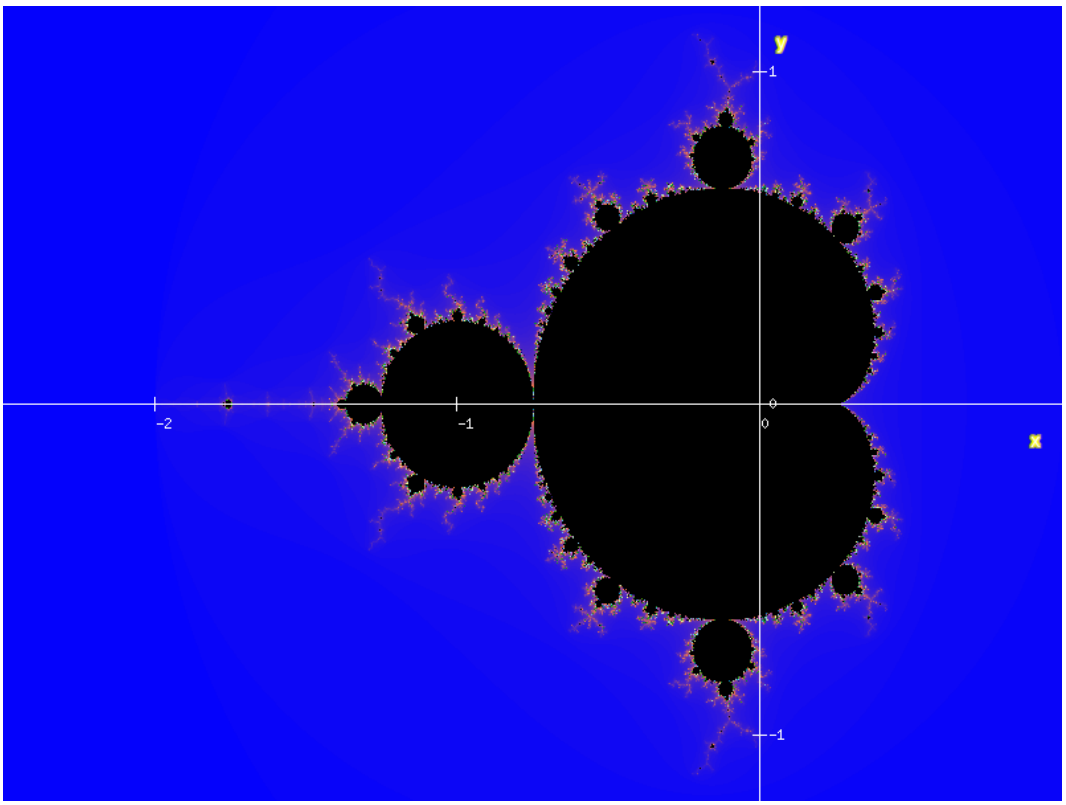

Figure 6: The Mandelbrot set in the Euclidean vector space

9 Conclusion

So we have seen that we can construct all the functions of complex analysis and the Mandelbrot set in the Euclidean vector space. In analogous ways we can continue with limits, etc.

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