The knotted cords and the origins of the sexagesimal system

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Abstract

This paper presents an explanation of the origins of the sexagesimal system linked to the use of a knotted cord 12 lengths long closed to form a loop. We will see that this device has also important geometrical properties linked to the division of the circle in 360 equal parts. Historical and cultural arguments will also be given to support this thesis.

1 Introduction

Several millenials ago, humans began to group elements in order to count them (see [Britan]). In time, humans developped regular systems as base 10, base 20, etc. And, in the area of Mesopotamia, the base 60 was developped. The history of sexagesimal covers several millenials (see [Friberg19]).

However, the question of the origin of the choice of 60 as a base has remained, until now, open, even if we know the advantages[1] of 60, for example the fact that it is divided by 2, 3, 5, 12, 30, etc. For other bases as 5 or 10, for example, it's quite clear that they are linked to the number of human fingers (see [Ifrah2000], p. 47).

In the following sections we will present a device, a knotted cord 12 lengths long closed to form a loop, which has important properties linked to base 60. It has also geometrical properties related to the circle and the division in 360 equal parts. We will show that this device is linked to the origins of sexagesimal. Historical and cultural arguments will also be given to support this thesis.

In this paper we are presenting only the origins of the base 60. The base 60, as it was employed in Mesopotamia, had also remains of the base 10. There was a special sign for 10, for exemple.

2 The knotted cord with 12 equally spaced knots and the base 60

At the beginning of his book "The universal history of numbers" [Stillwell2010], John Stillwell recalls the importance of the Pythagorean theorem. He writes: "The Pythagorean theorem is the most appropriate starting point for a book on mathematics and its history. It is not only the oldest mathematical theorem, but also the source of three great streams of mathematical thought: numbers, geometry, and infinity.".

After presenting the Pythagorean triples and their links with geometry, he writes: "It is thought that in ancient times such solutions may have been used for the construction of right angles. For example, by stretching a closed rope with 12 equally spaced knots one can obtain a (3, 4, 5) triangle with right angle between the sides 3, 4 (...)". And he presents a triangle as in figure 1.

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[1]There are other advantages of the sexagesiamal system. I wrote a paper about a geometrical link between the circle and sexagesimal system (see [Torres-Heredia2005] published in july 2005, in French, and [Torres-Heredia2007] published in july 2007 in English).

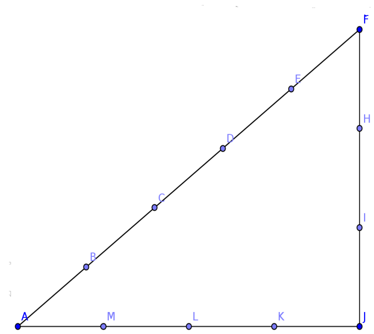

Figure 1: The knotted cord with 12 equally spaced knots

Now, we can remark that, if we multiply the three numbers corresponding to the sides, we get:

$$3 \cdot 4 \cdot 5 = 60$$

In fact, from the three sides of that triangle we can get the prime-power factorization of 60:

$$2^2 \cdot 3 \cdot 5 = 60$$

Moreover, with the length of the three sides we can construct a rectangular parallelepiped whose volume is 60, as in figure 2.

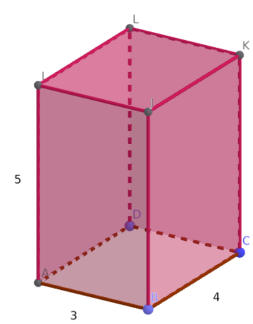

Figure 2: A rectangular parallelepiped whose volume is 60

This is important because the volume 60 is linked to that basic triangle. The base of that rectangular parallelepiped is formed of two times the basic triangle seen above. And in that rectangular parallelepiped we can, obviously, place 60 elements. That is related to the notion of capacity in a container. This can lead also to grouping elements in groups of 60 elements.

What we are seeing is that the number 60 is naturally linked to a basic triangle whose sides are the numbers of the simplest Pythagorean triple: (3, 4, 5). We are seeing also that this triangle can be constructed with a knotted cord with 12 equally spaced knots. We are seeing that with the lengths of the triangle we can construct a rectangular parallelepiped whose volume is 60.

So, those are some very good reasons to choose 60 for a base. In fact, besides the advantages mentioned above (60 is divided by 2, 3, 5, 30, etc.), 60 is linked to a very basic triangle consctructed with a knotted cord.

Furthermore, it is important to recall that, with the knotted cord with 12 equally spaced knots, we can form a circle as in figure 3. That circle is divided in 12 equal parts.

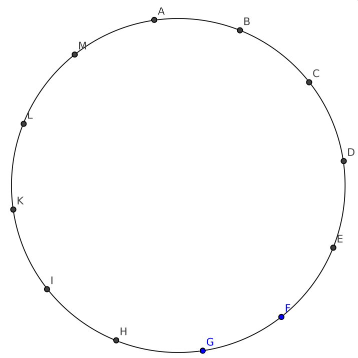

Figure 3: Circle divided in 12 equal parts.

Now, we know that if (a, b, c) is a Pythagorean triple, so is (ka, kb, kc) with $k \in \mathbb{N}, k \geq 2$. If $k = 30$ and $(a, b, c) = (3, 4, 5)$ we get $(90, 120, 150)$. So we get a triangle as in figure 4. So we get a triangle as in figure 4.

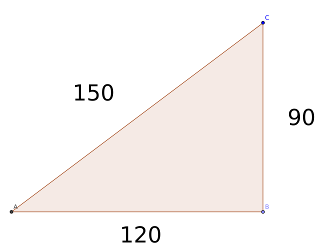

Figure 4: A triangle with the triple (90, 120, 150)

We can remark that:

$$90 + 120 + 150 = 360$$

So we can have also a knotted cord with 360 equally spaced knots. This fact could have also lead to to division of the circle into 360 equal parts. Another reason, among others, is that it is close to the 365 days of the year.

By the way, if $k = 2$ and $(a, b, c) = (3, 4, 5)$ we get $(6, 8, 10)$. We can remark that:

$$6 + 8 + 10 = 24$$

So we can get also a circle divided into 24 equal parts and it correspond to the number of hours of the day.

3 Historical and cultural arguments for the choice of base 60 from a knotted cord

In the previous section we have seen how the the knotted cord with 12 equally spaced knots is related to base 60 and the division of the circle into 360 equal parts, through Pythagorean triples.

Now, how can we prove that it was really used as this in Mesopotamia?

We will present several arguments:

1) It is known that Egyptians used knotted cords to perform measurements (see [Obenga1995], p. 22).

2) Acording to Plutarch, quoted by Roger Cooke (see [Cooke2011], p. 238), the triangle whose sides are 3, 4 and 5, was very regarded above all others by the Egyptians.

3) Roger Cooke gives some arguments that suggest that the Egyptians knew the Pythagorean theorem (see [Cooke2011], p. 238 and [WikipediaK]).

4) The Egyptian civilization was contemporary to the mesopotamian civilizations (Sumer, etc.) and was not very far from them.

6) The Babylonians knew Pythagorean triples around 1800 BC. We can see it in the Plimpton 322 clay tablet (see [WikipediaP]).

7) Roger Ifrah has recalled that knotted cords have been used for counting all around the world for millenials (see [Ifrah2000], pp. 67-72). Among the most known, there the quipus of the Inca Empire.

8) According to Bartel Leendert van der Waerden, in his "Geometry and Algebra in Ancient Civilizations" book, the integer-sided right triangles are ubiquitous in the oldest megalithic structures (see [Cooke2011], p. 238).

Given those 8 arguments, it is unlikely that those who had the idea of choosing 60 as a base, in Mesopotamia, didn't know all the facts linked to the knotted cord with 12 equally spaced knots and to the Pythagorean triples. And we know that there was also the division of the circle in 360 equal parts in Mesopotamia.

It's important to recall that in Ancient Civilizations, the numbers were used not only for calculations. They had also metaphysical meanings, as for Pythagoras. We have seen that the knotted cord with 12 equally spaced knots unifies geometry, arithmetic and even astronomy. That idea of unification was surely important.

4 Conclusion

So we have found a device that unifies geometry and arithmetic. The number 60 is naturally linked to a basic triangle whose sides are the numbers of the simplest Pythagorean triple: (3, 4, 5). This triangle can be constructed with a knotted cord with 12 equally spaced knots. And it is related to the origins of sexagesimal system, as it has been shown.

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[WikipediaP] Wikipedia, *Plimpton 322*, page consulted on July 2025.